CM

At time $t = t_0$, a starship leaves Earth. It travels directly away at a constant velocity $\beta = v/c = 0.8$, as measured in the frame of the Earth. At t_1 a hostile force on Earth fires a packet of 10^{12} radioactive ions at the escaping ship. Each ion has mass m_0. At t_2 a laser pulse is sent from Earth to warn the starship of the impending threat from the ions. The warning is received at t_3 —exactly at the moment that the ion packet also arrives.

Note: In this problem primed quantities are measured in the frame of the ship, and unprimed quantities such as t_1, t_2, t_3 refer to the reference frame of the Earth.

a) The momentum of each ion in the packet is $p_{\text{ion}} = 3m_0c$, What is the total energy E of the ion packet?

b) Let t_1 be 15 hrs. Find t_2 (when the laser is fired) and t_3 (the moment that the laser beam and ion packets simultaneously arrive at the ship).

c) The radiocative ions have a proper lifetime of 10 hrs. Assume that the ions that decay miss the rocket, while those that survive are fully absorbed. *In the reference frame of the starship*, how much total energy E' is absorbed by the ship?

d) A mirror at the back of the starship reflects part of the laser beam back to the earth. The wavelength of the light emited by the laser is 337 nm. Derive the relativistic Doppler shift starting from the Lorentz transformation and apply your result to determine the wavelength of the reflected light as it would appear from Earth.

You may find the following example Lorentz transformation to be useful:

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}, \quad \beta = v/c, \quad \gamma = (1-\beta^2)^{-1/2}.$$