The three parts of this problem are independent.

(a) A mass $m$ moves in a circular orbit of radius $r_0$ under the influence of a central force whose potential is $-km/r^n$ with $k > 0$. Show that the circular orbit is stable under small perturbations if $n < 2$.

(b) A yo-yo rests on a level surface. A gentle horizontal pull (see figure) is exerted on the string so that the yo-yo rolls without slipping.
   (i) Which way does it roll?

   (ii) The inner shaft of the yo-yo has one third the radius of the outer disk, which is $R$, and the moment of inertia of the yo-yo about its center is $I = \frac{1}{2}MR^2$ where $M$ is the mass of the yo-yo. What is the torque about the center of the yo-yo? Express your answer in terms of $F$, $M$, $R$, the acceleration due to gravity $g$ and the coefficient of friction $\mu$.
   (iii) Use the no-slip condition to obtain the (linear) acceleration in terms of $F$.
   (iv) What is the minimum value of the coefficient of friction, $\mu_{\text{min}}$, such that the yo-yo will roll without slipping?

(c) Consider a relativistic 1D harmonic oscillator: a particle of rest mass $m$ moving in a potential $\frac{1}{2}m\omega^2x^2$. Use conservation of energy to solve for the velocity $v(x)$. Argue that the first order relativistic correction to the period of the oscillator is

$$\tau = \frac{2\pi}{\omega} \left[ 1 + (\text{const.}) \frac{\omega^2a^2}{c^2} + \cdots \right]$$

where $a$ is the amplitude of the oscillator and $c$ is the speed of light. Is the constant positive or negative, i.e. does the period get longer or shorter once relativistic effects are included?