A solid cylinder of radius \( a \) and mass \( M \) rolls along a horizontal rail without slipping. The center of the cylinder is attached to the walls by four massless springs, each having a spring constant \( k \). A rod of negligible radius and mass \( m \) is hung from a frictionless and massless axle at the center of the cylinder by means of rigid, massless wires of length \( L \). The rod is free to swing in response to gravity as shown above. The moment of inertia of the cylinder about the axle is \( I = \frac{1}{2} Ma^2 \).

(a) Write down the kinetic energy \( T \) of this system in terms of the parameters given.

(b) Write down the potential energy \( V \) of this system in terms of the parameters given.

(c) Write down the coupled equations of motion describing the cylinder's displacement from equilibrium along the rail, \( X \), and the hanging rod's angle from equilibrium, \( \theta \). Do not attempt to solve or simplify the equations at this point.

[continued on next page]
(d) After making both of the following two simplifying assumptions to your answer in part (c), determine the frequencies of the normal vibrational modes of this system:

(i) Let \( M = 2m \) and \( kL = mg \), where \( g \) is the acceleration due to gravity, and

(ii) assume both the pendulum and cylinder have small oscillations.

(e) Assume that \( X(t = t_0) = X_{\text{max}} \), where \( X_{\text{max}} \) is the maximum positive displacement of the cylinder from equilibrium. For each of the normal vibrational modes, sketch in your answer booklet both the normalized cylinder displacement, \( X/X_{\text{max}} \), and the normalized rod swing angle, \( \theta/\theta_{\text{max}} \), as a function of time using the same time axis for both quantities. Sketch only for one full period \( T \) of the system's motion (see graph below).

![Graph showing normalized cylinder displacement and normalized rod swing angle as a function of time.]

(f) Identify which of the normal modes in part (e) has the lower frequency.