Two equal point masses $m$ hang from springs obeying Hook's law as shown in the figure below. The spring constant of each spring is $k$. The masses of the springs are negligible. The length of each spring in the relaxed state is $\ell$. The gravitational acceleration $g$ is pointing vertically down in the figure. The two masses are constrained to move only along the vertical direction (i.e. the motion is one-dimensional).

(a) Find the equilibrium positions of the masses, $x_{10}$ and $x_{20}$ in terms of $g$, $k$, $m$, and $\ell$.

(b) How many normal modes are there?

(c) Find the frequencies of the normal modes.

(d) In the figure below, the arrows indicate the direction of motion of one of the normal modes, which we call normal mode 1. In your exam booklet, draw a corresponding diagram for any other normal mode, indicating whether its frequency is higher or lower than that for the one shown.

(e) Calculate the normal modes.