A bead of mass \( m \) slides in a frictionless hollow open-ended tube of length \( L \) which is held at an angle of \( \beta \) to the vertical and rotated by a motor at an angular velocity of \( \omega \) about its lower endpoint. The apparatus sits in a uniform vertical gravitational field.

(a) Find the equation of motion of the bead in the tube.

(b) The bead is in equilibrium in the tube. Find the relationship between the position of the bead and the angular velocity of the tube.

(c) What is the minimum value of the angular velocity such that the bead remains at an equilibrium position inside the tube?

(d) The bead is very slightly displaced along the tube by a distance \( \delta \) from an equilibrium position and released from rest. Find the position of the bead as a function of time while it is still in the tube.

(e) The bead is at an equilibrium position of part (b). Suppose the bead is again very slightly displaced along the tube by a distance \( \delta \). At exactly the same time that the bead is released from rest, the motor is switched off and the tube allowed to spin freely without friction. Determine the position of the bead as a function of time while it is still in the tube. (Assume that the tube has negligible mass.)