Crispring96A

A rocket plane travels through still air in level flight, so that its weight is exactly compensated by the lift from the wings. It expels matter (i.e. loses mass) at a constant rate $\gamma$, with the speed of the exhaust relative to the rocket plane being $V$. The rocket plane experiences a drag force $\mathbf{F}_D$ from the air, which is proportional to its velocity $\mathbf{v}$ in the rest frame of the air (and also the ground): $\mathbf{F}_D = -k \mathbf{v}$. At time $t = 0$ the rocket plane has mass $m_0$, and is released from a jumbo jet flying horizontally with speed $v_0$.

(a) Determine the differential equation governing the speed $v(t)$ of the rocket plane.
(b) For the case where $k = 0$, determine $v(t)$ in terms of the parameters given above.
(c) For the case where $k \neq 0$, determine $v(t)$ in terms of the parameters given above.
(d) Verify that your solution to part (c) reduces to that of part (b) as $k \to 0$.
(e) For the case where $k \neq 0$, determine the general relationship among the parameters given above that permits the rocket plane to move at constant speed.