Consider the reaction, \( n + p \rightarrow d + \pi^0 \), in which a neutron interacts with a target proton to produce a deuteron and a \( \pi^0 \). Assume that the target proton is at rest in the lab frame. For this problem use: \( m_n = m_p = M \), \( m_d = 2M \), and \( m_{\pi} = m \).

(a) The neutron threshold momentum is the minimum neutron momentum needed for the reaction to occur. Derive an expression for the neutron threshold momentum, \( p_n \), in terms of \( M \) and \( m \).

(b) Derive expressions for the momentum of the \( \pi^0 \), and of the \( d \), at threshold. Leave your expression in terms of \( p_n \), the neutron threshold momentum that you found in part a).

(c) Assume that the \( \pi^0 \) created at threshold now decays immediately after its production into two \( \gamma \) rays, \( \pi^0 \rightarrow \gamma + \gamma \) (see diagram below). By considering conservation of momentum, derive an expression for the minimum possible angle \( \theta \) between the \( \gamma \)-rays in the lab frame, in terms of \( p_\pi \) and \( E_\pi \).