An initially uniform electric field, \( \vec{E}(\vec{r}) = E_o \hat{z} \) exists in between two infinitely large, parallel conducting plates, separated by a small vertical distance, \( d \). The lower plate is at ground potential (0 volts) and the upper plate is held at a constant potential \( -V_o \). A hemi-spherical dielectric, with dielectric constant \( K \equiv \varepsilon / \varepsilon_o > 1 \) and radius \( R << d \) is placed on the lower plate with its center at the origin of coordinates of the parallel-plate capacitor as shown in the above figure.

(a) State the boundary conditions for \( E_r(\vec{r}) \) and \( D_r(\vec{r}) \) at the surface of the hemi-spherical dielectric.

(b) Calculate the potential inside (\( \Phi_{\text{in}}(\vec{r}) \)) and outside (\( \Phi_{\text{out}}(\vec{r}) \)) the hemi-spherical dielectric.

**Hint:** You may use the following formula (without derivation):

\[
\Phi(\vec{r}) = \sum_{l=0}^{\infty} \left( a_l r^l + b_l \frac{1}{r^{l+1}} \right) P_l(\cos \theta)
\]

where \( r \) and \( \theta \) are spherical polar coordinates.

(c) Calculate the electric field inside (\( E_{\text{in}}(\vec{r}) \)) and outside (\( E_{\text{out}}(\vec{r}) \)) the hemi-spherical dielectric.

(d) Make a detailed, careful sketch of the electric field lines and the equipotentials (lines of constant potential) between the parallel plates, and also inside and outside the hemi-spherical dielectric. Draw the electric field lines as **solid** lines and the equipotentials as **dashed** lines.

(e) Calculate the bound charge density, \( \sigma_{\text{bound}} \) on the upper surface of the hemi-spherical dielectric. Express your answer in terms of \( E_0 \equiv V_o / d \), \( K \) and \( \theta \).

\[
\sigma_{\text{bound}} = \frac{1}{\varepsilon_o} \oint_{\text{surface}} \vec{E}(\vec{r}) \cdot d\vec{A}
\]