A 1.5 V battery is connected with a switch to uncharged circular capacitor plates of radius \( r = 10 \text{cm} \). The plates are \( d = 3.1 \text{mm} \) apart. Two long metal rods hold the plates in place and carry the current to the plates. The internal resistance of the battery is 10 Ohms. Ignore fringe fields.

a) Model this system as an RC circuit and determine the time dependence of the displacement current between the plates after the switch is closed. Calculate the capacitance. Sketch the time dependence of the current. What is the time constant \( \tau \) of the circuit (the time when the current has dropped to \( 1/e \) of its initial value)?

b) Use a coordinate system where the \( z \)-axis is along the symmetry axis of the capacitor and the origin is at the center of the capacitor. Determine the time dependence of the magnitude of the electric field at the position (2 cm, 0, 0) between the plates, and at (20 cm, 0, 0) outside the plates for \(- \tau < t < 2 \tau \). State the direction of the electric field and plot the magnitude of the electric field on the \( x \)-axis between \( x = 2 \text{cm} \) and \( x = 20 \text{cm} \) at time \( t = \tau \).

c) Determine the time dependence of the magnitude of the magnetic field at the position (2 cm, 0, 0) between the plates, and at (20 cm, 0, 0) outside the plates for \(- \tau < t < 2 \tau \). State the direction of the magnetic field and plot the magnitude of the magnetic field on the \( x \)-axis between \( x = 2 \text{cm} \) and \( x = 20 \text{cm} \) at time \( t = \tau \).

d) Now suppose that the circuit has an inductance of 1 mH. Write down a differential equation that describes the dynamics of the charge on the capacitor. Is the resonance sharp or overdamped. Justify your answer.