A two dimensional gas of electrons is confined at \( z = 0 \) in the \( x-y \) plane at the interface between two semi-infinite dielectric slabs, as shown above. Each slab has dielectric constant \( \epsilon \). A perturbation of the electron charge-density propagates through the electron gas. The total two dimensional charge density is given by \( \sigma(x, t) = \sigma_0 + \delta \sigma(x, t) \), where the perturbation \( \delta \sigma \) takes the form of a wave

\[
\delta \sigma(x, t) = a \exp\{i(kx - \omega t)\}.
\]

You may work in either SI or CGS units and assume that:

- Electrons act as classical particles of mass \( m \) with local velocity,

\[
v(x, t) = v_0 \exp\{i(kx - \omega t)\}.
\]

- Magnetic fields are negligible.
- The perturbation is small, \( i.e. \delta \sigma \ll \sigma_0 \).

Now do the following:

a) Use Laplace’s equation to find the electrical potential \( \phi(x, z, t) \) due to the periodic charge perturbation.

b) From \( \phi(x, z, t) \) find the electric field component \( E_x(x, z = 0, t) \) parallel to and within the electron gas.

c) Use the linearized charge continuity equation to find a relation between \( a \) and \( v_0 \).

d) Show that the relation between \( \omega \) and \( k \) for the wave is given by,

\[
\omega^2 = \gamma |k|,
\]

where you should express the coefficient \( \gamma \) in terms of \( m, \epsilon, \sigma_0 \) and the electron charge \( q = -e \).