Two perfectly conducting metallic hemispherical shells of radius $a$ are separated by a very small insulating gap and form a sphere of radius $a$, as shown.

(a) Suppose the top hemisphere has static potential $+V_0$ and the lower hemisphere has static potential $-V_0$. Calculate the electric dipole moment, $p_0$.

Work parts (b) and (c) to lowest nonvanishing order in $\omega$:

(b) Suppose the potentials oscillate at low frequency, so that the top shell has potential $+V_0 \cos(\omega t)$ and the bottom shell has potential $-V_0 \cos(\omega t)$. Sketch the pattern of the radiated power as a function of the polar angle, $\theta$.

(c) Calculate the time averaged power radiated per unit solid angle as a function of $a$, $\omega$, $V_0$, $\theta$, and appropriate constants. If you have forgotten the formula for radiated power, a correct dimensional analysis will earn you nearly full credit.

(d) For what values of $\omega$ is it a good approximation to consider only the lowest nonvanishing order in $\omega$?