An insulating sphere of radius $R$ contains a fixed spherically symmetric distribution of electric charge with density $\rho(r)$. The form of the function $\rho(r)$ as a function of radius $r$ is not specified. The sphere rotates rigidly with angular frequency vector $\Omega = \Omega \hat{z}$, $\Omega > 0$.

A) Give an expression for the electrostatic potential $\phi(r)$ at any point $r$ in terms of an integral involving $\rho(r)$. The potential is zero at infinity.

B) Give an expression for the current density vector $j(r)$ in terms of the quantities given, at any point inside the sphere.

C) Give an expression for the magnetic field vector $B(r)$ at any point $r$ in space in terms of an integral over the current density found in part (B).

D) Find a simple expression for the ratio $\frac{|B(0)|}{\phi(0)}$ in terms only of $\Omega$ and fundamental constants.

E) Give the direction of $B(0)$ if the charge density is positive everywhere.

HINT: You may find it convenient to use the vector relation

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$$