This problem illustrates the concept of magnetic mirrors for neutral atoms. Consider an infinite sheet of magnetic material placed in the x-z plane in the form of a magnetic recording tape. The tape has a nonuniform periodic magnetization \( M = \hat{x} \cos(2\pi x/\lambda) \), where \( \lambda/2 \) is the distance between the north (N) and south (S) poles of the magnetization. The region outside the tape is a vacuum. Assume that there are no real currents or time varying fields anywhere in space.

Assume that the top surface of the tape is at \( y = 0 \), and that the B-field at the top is given by \( B = B_0[\hat{y} \sin(2\pi x/\lambda) - \hat{x} \cos(2\pi x/\lambda)] \). State the units that you are using to solve this problem.

[continued on next page]
(a) Show that the $H$ field in the vacuum region above the tape may be determined from a magnetic scalar potential $\Phi_m(x, y)$, such that $H = -\nabla \Phi_m$ in analogy with electrostatics. Show that $\Phi_m$ satisfies a two-dimensional Laplace's equation.

(b) Determine the most general solution for $\Phi_m(x, y)$ by solving Laplace's equation.

(c) State the boundary conditions on $B$ and $H$, at $y = 0$ and $y = \infty$, that are needed to evaluate $B$ in the vacuum region.

(d) Use the boundary conditions to evaluate $B$ in the vacuum.

(e) An ultra-cold hydrogen atom of magnetic moment $-\mu \hat{y}$ approaches the top surface of the tape from above and is traveling initially in the $-\hat{y}$ direction along the line $x = 3\lambda/4$. Determine the force exerted by the tape on the atom as a function of distance from the tape. Does the tape attract or repel this atom?