An infinite lossless coaxial transmission line is characterized by its inductance and capacitance per unit length, $L_0$ and $C_0$. The figure below represents a model of such a transmission line; here $L = L_0 \Delta x$ and $C = C_0 \Delta x$. The voltage and current are described by functions $V(x,t)$ and $I(x,t)$.

(a) Show that the transmission line is described by the partial differential equations

$$\frac{\partial V}{\partial x} = -L_0 \frac{\partial I}{\partial t}$$

and

$$\frac{\partial I}{\partial x} = -C_0 \frac{\partial V}{\partial t},$$

which can be obtained by taking the continuum limit $\Delta x \to 0$ in the circuit above.

(b) Find the form of the traveling-wave solutions to the partial differential equations in part (a).

(c) Determine the velocity of propagation of waves on the transmission line in terms of the parameters of the line.

(d) Determine the characteristic impedance of the line, i.e., $Z_0 = V / I$.

(e) Now suppose that the line is terminated with a load of impedance $Z_L$. Show that the ratio of the reflected voltage wave to the incident voltage wave is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}.$$