In the above figure, an incident plane wave travelling in a nonconductive medium of permittivity \( \varepsilon \) and permeability \( \mu \) is totally reflected at the interface with vacuum at \( x = 0 \). The incident and the reflected wave vectors lie in the xy-plane,

\[
\vec{k}_I = (k_I \cos \theta_I, k_I \sin \theta_I, 0) \\
\vec{k}_R = (-k_R \cos \theta_R, k_R \sin \theta_R, 0)
\]

Consider the case that the polarizations described by the E-fields of the incident wave, \( E_I \), the reflected wave, \( E_R \), and the wave in the forbidden regions, \( E_T \), are all in the z-direction,

\[
\vec{E}_I = (E_I)_0 e^{i \vec{k}_I \cdot \vec{r} - i \omega t} \hat{z}
\]

\[
\vec{E}_R = (E_R)_0 e^{i \vec{k}_R \cdot \vec{r} - i \omega t} \hat{z}
\]

and

\[
\vec{E}_T = (E_T)_0 e^{ -kx + i k_y y - i \omega t} \hat{z}
\]

In the forbidden region (\( x > 0 \)), the E-field is a wave that attenuates in the x-direction, and propagates in the y-direction.

(a) Write down (but you need not derive) the partial differential equation satisfied by the \( \vec{E} \) field in the allowed (\( x < 0 \)) and forbidden (\( x > 0 \)) regions, in terms of \( \vec{E} \), \( x \), \( t \), \( v \) and \( c \). Here

\[
v = 1/\sqrt{\varepsilon \mu} \quad c = 1/\sqrt{\varepsilon_0 \mu_0}
\]

are the speed of light in the medium and vacuum respectively.

(Cont.)
(b) Hence, show that
\[ \omega^2 = k_I^2 v^2 = k_R^2 v^2 \]
and
\[ \omega^2 = c^2 (k_T^2 - \kappa^2) \]

(c) By matching the y-dependence of the three waves, obtain \( \kappa \) and \( k_T \) as functions of \( k_I, \theta_I \), and the index of refraction \( n = c/v \). Find the restriction on \( \theta_I \) and \( n \) such that real \( \kappa \) and \( k_T \) solutions exist.

(d) Obtain \( \vec{B}_T \) in the forbidden region \( (x > 0) \). Express your answer in terms of \( \kappa, k_T, \omega, \) and \( \vec{E}_T \) \( 0 \).

(e) Obtain \( \vec{B}_I \) and \( \vec{B}_R \) in the allowed region \( (x < 0) \) in terms of \( k_I, \theta_I, (E_I)_0, \) and \( (E_R)_0 \).

(f) State the boundary conditions at the interface \( x = 0 \) that \( \vec{E} \) and \( \vec{B} \) must satisfy.