Consider a system of two distinguishable particles, each with spin \( \frac{1}{2} \hbar \). In this problem all degrees of freedom other than the spins are to be ignored. Let \( \hat{s}_1 \) and \( \hat{s}_2 \) be the vector operators for spins of the particles. Let the Hamiltonian of this system be

\[
\hat{H} = A \hat{s}_1 \cdot \hat{s}_2,
\]

where \( A \) is a constant.

(a) Calculate the energy spectrum of this system.

Now, assuming at \( t = 0 \), particle 1 has spin up \( s_{1,z} = \frac{1}{2} \hbar \) and particle 2 has spin down \( s_{2,z} = -\frac{1}{2} \hbar \).

(b) Express the wavefunction of this system at \( t = 0 \) in terms of the eigenstates of the Hamiltonian.

(c) At \( t > 0 \), find the probability for particle 1 to have spin up \( s_{1,z} = \frac{1}{2} \hbar \).

(d) At \( t > 0 \), find the probability for the total spin of the system, \( S \), to have a value of \( \hbar \).

(e) At \( t > 0 \), what are the expectation values for \( s_{2,z}, s_{2,x}, \) and \( s_{2,y} \)?