A isolated light-responsive molecule has an excited state whose energy above the ground state is $E_0$. Consider two of these molecules that are no longer isolated, but lie close to each other, being separated by a small distance $R$. We will label one of them $D$ for donor, and the other $A$ for acceptor. We consider only the states $|1\rangle \equiv |D^*, A\rangle$ and $|2\rangle \equiv |D, A^*\rangle$ of the two-molecule system. Here the * denotes that the labelled molecule is in its excited state. The Hamiltonian for the combined two-molecule system has matrix elements

$$
\langle 1|H|1\rangle = \langle 2|H|2\rangle = E_0, \quad \langle 1|H|2\rangle = \langle 2|H|1\rangle = V.
$$

All other states of the system, and any matrix elements between them, can be ignored.

a) Find the energy eigenvalues of the two-molecule system. Express your answer in terms of $E_0$ and $V$.

b) Find the energy eigenstates of the two-molecule system as a linear combination of $|1\rangle$ and $|2\rangle$.

c) At time $t = 0$, only the donor molecule is excited. The two-molecule system is therefore in state $|1\rangle$. What is the earliest time $t$ at which the excitation energy is entirely transferred to the acceptor molecule $A$?

d) The off-diagonal matrix element

$$
V = \langle 2|H|1\rangle = \langle D, A^*|H|D^*, A\rangle
$$

arises from the dipole-dipole interaction between the molecules, and is given (in SI units) by

$$
V = \langle D, A^*|H|D^*, A\rangle = \frac{\mu_D \cdot \mu_A}{4\pi\varepsilon_0 R^3} = \frac{e^2}{4\pi\varepsilon_0 R^3} \langle D|r_D|D^*\rangle \cdot \langle A^*|r_A|A\rangle.
$$

In $\mu_D = e\langle D|r_D|D^*\rangle$ the vector $r_D$ is the displacement of an electron from the center of charge of molecule $D$. Similarly $r_A$ is the displacement of an electron in molecule $A$. The “.” in the rightmost expression indicates a scalar product between the two displacement vectors. It is reasonable to conjecture that $|\mu|/e$ is smaller than the size of the molecule. Given that the energy transfer between the donor $D$ and acceptor $A$ takes about 1 ns when $R = 5$ nm, use your answer to part (c) to estimate $|\mu|/e$ and compare it with the size (roughly 1 nm) of a light-responsive molecule. Is the conjecture about the magnitude of $|\mu|/e$ supported by your estimate?