**QM** This problem relates to some quantum aspects of muon catalysis of the fusion reaction $d + t \to \alpha + n$.

a) Explain (no derivation needed) what needs to be done to the textbook hydrogen-atom problem with $V(\mathbf{r}) = -ke^2/|\mathbf{r}|$ (where $k$ depends on your choice of units) to take into account that the electron and nucleus **both move** about their common center of mass. Recall that for the textbook fixed-nucleus problem the hydrogen ground-state energy is $E_0 = -13.6\,\text{eV} = -m_e k^2 e^4/2\hbar^2$.

b) A $\mu^-$ can bind with any of the nuclei $p$, $d$, $t$ to form a muonic atom. Which one will have the lowest ground-state energy?

c) Compute the energy difference (in eV's) between the ground states of the muonic atoms $d\mu^-$ and $t\mu^-$.

Now suppose that a muonic molecule composed of the three particles $dt\mu^-$ is formed. Since the $\mu^-$ is significantly lighter than either of the nucleons, in this part of the question we will **ignore the nuclear motion** and use the Born-Oppenheimer approximation in which the nuclei are held fixed a distance $R$ apart, and a Schrödinger equation is solved to find the ground-state energy $E(R)$. The actual nuclear separation will be that $R$ which makes $E(R)$ a minimum.

d) Write down the Schrödinger equation that must be solved to find $E(R)$.

e) Let $\psi_0(\mathbf{r})$ be the minimum-$E(R)$ ground-state eigenfunction for the electron in the $dte^-$ molecule. Adapt the same function $\psi_0$ to solve the $dt\mu^-$ problem. Hence find the ratio of bond lengths and the ratio of the chemical binding energy $E_{\text{binding}} = \min\{E(R)\}$ for the $dt\mu^-$ molecule compared to the $dte^-$ molecule.

f) For the fusion reaction to occur the two nuclei, $d$ and $t$, must be in the total $J = 3/2$ nuclear spin state. Assuming that the spins of the two nuclei are initially unpolarized, what is the probability that $J = 3/2$?

| particle | | charge | spin | mass (MeV) |
|---|---|---|---|---|
| electron | $e^-$ | -1 | 1/2 | 0.511 |
| muon | $\mu^-$ | -1 | 1/2 | 106 |
| proton | $p$ | +1 | 1/2 | 938 |
| deuteron | $d$ | +1 | 1 | 1876 |
| triton | $t$ | +1 | 1/2 | 2791 |