The deuteron is a weakly bound state of a neutron and a proton. The nuclear interaction between the proton and neutron can be approximated by a three-dimensional square well potential in the center-of-mass frame:

\[ V(r) = \begin{cases} 
-V_N & r < r_N = 10^{-13}\text{cm} \\
0 & r > r_N 
\end{cases} \]

where \( r \) is the separation between the proton and the neutron at positions \( \vec{r}_p \) and \( \vec{r}_n \) respectively. In the center-of-mass frame the two-body problem reduces to that of a single particle with reduced mass \( \mu \) moving in the potential well \( V(r) \).

(a) Derive the reduced mass \( \mu \) in terms of the neutron and proton masses, \( m_n \) and \( m_p \) and find the radial Schrödinger equation for this system.

(b) Assuming a bound state exists, what is the angular momentum of the ground state? Find the ground state wave function in the center-of-mass frame.

(c) Whether or not the neutron and the proton form a bound state depends on \( V_N \) and \( r_N \). Determine \( V_{\text{min}} \), the minimum value of \( V_N \) for a bound state to exist, in terms of \( r_N \) and \( \mu \). Calculate \( V_{\text{min}} \) in MeV using \( m_p = 938 \text{ MeV}/c^2 \) and \( m_n = 940 \text{ MeV}/c^2 \). (Note: \( \hbar c = 1.9732710^{-11}\text{MeV cm} \))

(d) Suppose that \( V_N \) is changed by a fraction \( \delta \): \( V_N = V_{\text{min}} (1 + \delta) \). Determine the ground state energy to leading order in \( \delta \) when \( 0 < \delta \ll 1 \).