Consider the three-level system defined by the Hamiltonian $H_0$ with distinct eigenvalues $\varepsilon_a$, $\varepsilon_b$ and $\varepsilon_c$, corresponding to the eigenstates $|A\rangle$, $|B\rangle$ and $|C\rangle$, respectively. This system is subject to a time-independent perturbation, $H_1$, with real matrix elements:

$$\langle A|H_1|A\rangle = \langle B|H_1|B\rangle = \langle C|H_1|C\rangle = 0$$

and

$$\langle A|H_1|B\rangle = \langle B|H_1|C\rangle = M; \text{ and } \langle A|H_1|C\rangle = 0$$

(a) Calculate the energy eigenvalues of the full Hamiltonian $H_0 + H_1$ to second order in $M$ (i.e., including terms of order $M^2$).

(b) Calculate the corrections to the state vectors $|A\rangle$, $|B\rangle$ and $|C\rangle$, to order $M$.

(c) Suppose now that the unperturbed states are degenerate, so that $\varepsilon_a = \varepsilon_b = \varepsilon_c = \varepsilon$. Calculate the eigenvalues and eigenvectors of the full Hamiltonian to leading order in $M$. 