Consider a plane rotator: two rigidly connected particles of mass $m$ rotating in the xy plane about their center of mass. The rod of length $d$ connecting the particles has negligible mass.

![Diagram of a plane rotator with particles at positions $m$ and $m$ connected by a rod of length $d$.]

(a) Write down the system's Hamiltonian in terms of $m$, $d$ and the angle $\varphi$.

(b) Suppose the initial state of the rotator is given by the wave function

$$\psi(\varphi, t = 0) = A \cos^2 \varphi$$

where $A$ is a constant. Consider a single measurement of the angular momentum $L_z$ in this state.

- What are the possible values of $L_z$ observed in that measurement?
- What are the probabilities for each value?
- What is the expectation value of $L_z^2$ in this state?

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(c) Find $\psi(\varphi,t)$ for $t > 0$, given the initial state in part (b). What is the angular frequency $\omega_1$ of the periodic time dependence of $\psi(\varphi,t)$?

(d) Suppose that at $t = t_0$, the distance $d$ between the particles of the plane rotator collapses suddenly to $d/2$. What is the new Hamiltonian for $t > t_0$? Solve for $\psi(\varphi,t)$ for $t > t_0$.

(e) What is the new angular frequency $\omega_2$ for $t > t_0$? How does the ratio $\omega_1/\omega_2$ compare to the same quantity calculated classically?