A simple model of a semiconductor is to assume that it has $M$ donor electron levels, all at a negative energy, $-\Delta$, and a free particle electron conduction band whose minimum energy is zero. Assume that at zero temperature the conduction band is empty, and that all the donor levels are filled, with one electron per donor. At finite temperature, $T$, a number $N$ of the electrons are thermally excited from the donor states to the conduction band, leaving behind $N$ holes in the donor levels. Assume $N \ll M$ throughout, and ignore electron-electron interactions. Let the volume of the semiconductor be $V$.

\begin{center}
\textbf{Conduction band}
\end{center}

\begin{center}
\begin{tikzpicture}
\draw (-1,0) -- (5,0);
\foreach \x in {0,1,2,3,4,5}
\draw[fill=black] (\x,0) circle (0.1cm);
\draw (5,0) -- (6,0);
\draw (-1,0) -- (-1,-0.5);
\draw (6,0) -- (6,-0.5);
\node at (2.5,-1) {$-E = 0$};
\node at (2.5,-1.5) {$-E = \Delta$};
\node at (0,-2) {$\text{Donor levels}$};
\end{tikzpicture}
\end{center}

a) Determine the energy, entropy and (Helmholtz) free energy of the electrons in the conduction band. In doing so, assume Maxwell-Boltzmann statistics and express the entropy in terms of $N$ and the “thermal wavelength,” $\lambda_{th} = (2\pi\hbar^2/mk_BT)^{1/2}$, where $m$ is the electron mass, and $k_B$ is Boltzmann’s constant. In terms of $\lambda_{th}$, what is the condition that the conduction electrons obey Maxwell-Boltzmann statistics?

b) Determine the total energy, entropy, and free energy of the $M - N$ electrons and $N$ holes in the donor levels? Assume $N \gg 1$ and use Stirlings approximation, $\ln N! \simeq N(\ln N - 1)$.

c) Determine the equilibrium number of electrons excited to the conduction band as a function of $M$, $T$, and $\Delta$, including the free energy of the electrons in the donor levels, but neglecting the entropy of the electrons in the conduction band.

d) Determine the equilibrium number of electrons excited to the conduction band as a function of $M$, $T$, and $\Delta$, including the free energy of the electrons in the conduction band, but neglecting the entropy of the electrons in the donor band.

e) Determine the equilibrium number of electrons excited to the conduction band as a function of $M$, $T$, and $\Delta$, taking into account all the terms in the free energy of the electrons in both the donor levels and the conduction band.