In this problem, we will be exploring the Bose-Einstein transition temperature, $T_{BE}$, of a gas of $N$ non-interacting spinless Bose particles. Below $T_{BE}$, there is macroscopic occupation of the ground state. Here, each of the particles has mass $m$ and all are enclosed in a three-dimensional volume $V$ at temperature $T$.

(a) Find an expression for the density of available single-particle states $D(\epsilon)$ as a function of the single-particle energy $\epsilon$. Sketch $D(\epsilon)$, being careful to label the axes and the origin.

(b) What is the allowed range of $\mu$ for a non-interacting Bose-Einstein gas?

(c) Write down an expression for the mean occupation number of a single particle state, $<n>$, as a function of $\epsilon$, $T$ and $\mu(T)$, where $\mu$ is the chemical potential and $T_{BE} < T < \infty$. Sketch $<n>$ at temperature $T$ as a function of $\epsilon$. Mark the location of $\mu$.

(d) Write down an integral expression which implicitly determines $\mu(T)$.

As the temperature, $T$, is lowered, how does $\mu(T)$ change? It will help to refer to your sketch.

(e) Using your answer to (d), find $T_{BE}$.

You may find this useful: 
\[
\int_0^\infty dx \frac{x^{1/2}}{e^x - 1} = 1.306 \pi^{1/2}
\]