Consider a point-mass \( m_1 \) located at \((x_1, 0)\) that can slide on a frictionless horizontal rail oriented along the \( x \) axis. A pendulum of mass \( m_2 \), located at \((x_2, y_2)\), is attached to \( m_1 \) with a massless rigid wire of length \( l \). The motion of the pendulum is confined to the \( x-y \) plane (see figure).

a) Take the position \( x_1 \) of \( m_1 \) and the angle \( \theta \) that the pendulum makes with the vertical as the two generalized coordinates. Write down the Lagrangian \( L \) of this system in terms of the generalized coordinates and their time derivatives.

b) From the Lagrangian \( L \), obtain the two equations of motion for the system.

c) Write down the momenta \( p_{x_1}, p_{\theta} \) that are canonically conjugate to your generalized coordinates and use them to obtain the Hamiltonian of the system. Do you expect its value \( E \) to be a conserved quantity? Explain why or why not.

d) Write the expression, in terms of the generalized coordinates and their time derivatives, for the total horizontal momentum \( P_x = m_1 \dot{x}_1 + m_2 \dot{x}_2 \) of the system. Is \( P_x \) a conserved quantity? Why or why not?

e) Similarly write down the vertical component of the momentum \( P_y = m_2 \dot{y}_2 \) in terms of the generalized coordinates. Is it a conserved quantity?