**EMA**: When a perfectly conducting sphere of radius \( R \) is placed in an initially uniform electric field \( \mathbf{E} = E_0\hat{z} \) it develops a surface charge distribution \( \sigma = 3\varepsilon_0 E_0 \cos \theta \) where \( \theta \) is the spherical polar coordinate.

![Diagram of sphere with electric field](image)

a) What is the total charge on the sphere?

b) Calculate the electric dipole moment \( \mathbf{p} \) of the sphere.

c) If we take the potential of the surface of the sphere to be zero, what is the potential \( \varphi(r) \) at all points \( r \) outside the sphere? Express your answer in terms of \( \mathbf{p} \). (Remember to include the potential of the field \( \mathbf{E} = E_0\hat{z} \). As a check of your computation verify that your expression for \( \varphi(r) \) is indeed constant everywhere on the sphere’s surface.)

We now build an artificial dielectric consisting of identical perfectly conducting metal spheres of radius \( R \) which are randomly distributed in an unpolarizable medium. The average number of spheres per unit volume is \( N \). Assume that when we insert the dielectric in a uniform field we can neglect the difference between the actual electric field acting on each sphere and the applied uniform electric field (i.e., the polarizability is weak).

d) Starting from an appropriate Maxwell equation, derive the relation between the electric field \( \mathbf{E} \), the electric displacement field \( \mathbf{D} \), and the polarization \( \mathbf{P} \) in a dielectric material.

e) Use your results from parts (b) and (d) to compute the dimensionless dielectric constant (also called the *relative permittivity*) of our artificial dielectric.