EMB: Consider an infinitely long transmission line consisting of two perfectly conducting wires running parallel to the $x$ axis at $y = 0$, and $y = a > 0$ and separated by a dielectric. The wires have capacitance $C$ per unit length and inductance $L$ per unit length. In this problem the longitudinal components $E_x$ and $B_x$ both vanish everywhere, so we are examining transverse electromagnetic (TEM) waves.

![Diagram of transmission line with wires at $y = 0$ and $y = a$.](image)

a) Let $V(x, t)$ be the position- and time-dependent EMF between the wires that is defined by evaluating

$$V(x, t) = -\int_{0}^{a} E_y(x, y, t) dy$$

along a straight path between the wires at fixed $x$. Show that

$$\frac{\partial V}{\partial x} = \kappa \frac{\partial I}{\partial t}$$

for some constant $\kappa$ depending on $L$ and/or $C$ that you should find. Here $I(x, t)$ is the current at $x$ in the lower ($y = 0$) wire. The current at $x$ in the upper ($y = a$) wire has the same magnitude, but opposite direction.

b) By using the charge continuity equation, derive a similar relation between $\partial V/\partial t$ and $\partial I/\partial x$.

c) Combine the differential equations from parts (a) and (b) to obtain a wave-equation for $I(x, t)$, and express the wave velocity $c_{TEM}$ in terms of $L$ and $C$.

d) If $I(x, t) = I_0 \sin(kx - c_{TEM}t)$ what is $V(x, t)$?

e) By what factor will the RMS power transmitted by the line change when $L$ is increased by a factor of 10 while keeping $I_0$ fixed.