QM

Consider a quantum simple harmonic oscillator with time-dependent frequency and Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m}{2}\omega^2(t)x^2.$$

Let

$$\omega^2(t) = \begin{cases} \omega_0^2, & t < 0 \\ \omega_0^2 + \omega_1^2(1 - \exp\{-t/\tau\}), & t > 0, \end{cases}$$

here ω_0, ω_1, τ are constants. For all $t < 0$ the oscillator is in its ground state $|0\rangle$.

a) Using the wavefunctions given below, compute the matrix elements $\langle 0|x^2|0\rangle$ and $\langle 2|x^2|0\rangle$ where $|n\rangle$ are the eigenstates of the *original* $\omega = \omega_0$ oscillator.

b) Suppose that τ is very large (the *adiabatic limit*). Compute, without making any approximations, the probabililty that the oscillator ends up in its *new* ground state $|0\rangle'$ at times $t \gg 0$.

c) Using the phase choice for the wavefunctions given below, does the adiabatically evolving state of part (b) accumulate any geometric phase?

d) Suppose now that $\tau = 0$ — *i.e.* the *sudden limit*. Compute, again without making any approximations, the probability that the oscillator is in its *new* ground state $|0\rangle'$ at $t \gg 0$.

e) When τ is neither large or small, use first order time-dependent perturbation theory to compute the the amplitude that the oscillator is to be found in its *original* $\omega = \omega_0$ ground state at time $t > 0$.

The normalized energy eigenfunctions for the $\omega^2 = \omega_0^2$ oscillator are

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} \exp\left\{-\frac{m\omega_0 x^2}{2\hbar}\right\} H_n\left(\sqrt{\frac{m\omega_0}{\hbar}}x\right),$$

You will only need

$$H_0(x) = 1, \quad H_2(x) = 4x^2 - 2.$$