Phase slips and vortex-antivortex pairs in superconducting films with constrictions

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(Dated: December 11, 2003)

A system of two thin coplanar superconducting films seamlessly connected by a bridge is studied. When such a system is cooled, two distinct resistive transitions are observed. The first one is identified with the Berezinskii-Kosterlitz-Thouless (BKT) transition in the film, whereas the second one is a crossover related to the quenching of thermally activated phase slips (TAPS) occurring on the bridge. Using direct and indirect methods, we have measured the resistance $R(T)$ of such samples over a range of eleven orders of magnitude. Over this broad range of resistance a good agreement is found with the Langer-Ambegaokar-McCumber-Halperin (LAMH) theory of TAPS, which was adopted for short bridges, as given by the simple expression $R(T) = R_n \exp \left[-(c/\tau)(1 - t)^{3/2}\right]$, where $\tau = T/T_c$ and $R_n$ is the normal resistance of the bridge.

Thermal fluctuations are known to generate a non-zero resistance in superconductors with reduced dimensionalities.\textsuperscript{1} In two-dimensional (2D) thin films the fluctuations which cause the dissipation are known to be broken vortex-antivortex pairs (VAP)\textsuperscript{2–10} while in one-dimensional (1D) wires the resistance is caused by thermally activated phase slips (TAPS).\textsuperscript{11–19} One important difference between these two types of fluctuations is that in 2D a topological phase transition takes place, i.e. the Berezinskii-Kosterlitz-Thouless (BKT) transition, while in 1D only a crossover occurs. The BKT transition takes place at a finite temperature, below which the VAP are bound and the sample resistance is zero. On the contrary, in 1D wires the resistance is always greater than zero because the TAPS has a finite probability at any $T > 0$.\textsuperscript{11} These 1D fluctuations are described by the theory of Langer-Ambegaokar-McCumber-Halperin (LAMH).\textsuperscript{12,13}

In this Communication we present an experimental study of the interplay between TAPS and VAP types of fluctuations in structures with a mixed dimensionality. For this purpose we fabricate and measure a series of thin superconducting films interrupted by constrictions or “bridges” (Fig. 1a). The width of the bridges is in the range 10–30 nm, a few times larger than the coherence length $\xi(0) \approx 7$ nm in MoGe. Two resistive transitions are observed in such samples, indicating that VAP and TAPS are two independent processes. For $T > T_{BKT}$ the contribution of VAPs is dominant. Below $T_{BKT}$ the transport properties are well described by the LAMH theory. Using different techniques we have measured the resistance variation over eleven orders of magnitude, without detecting any significant deviation from the LAMH.

Before presenting experimental results we give a brief review of the BKT and LAMH theories. The BKT theory applies to 2D films and predicts a universal jump in the superfluid density at the characteristic temperature $T_{BKT}$, which is lower than the mean field critical temperature of the film, $T_{c0}$. The voltage-current characteristics are described by a power law via $V \sim I^\alpha$. For $T < T_{BKT}$ the VAP are bound and therefore the $V(I)$ curves are nonlinear with $\alpha \geq 3$, meaning that the zero-bias resistance $R(T)$ is zero. At $T > T_{BKT}$ one expects $\alpha = 1$, corresponding to the occurrence of a non-zero resistance $R(T) > 0$. Above $T_{BKT}$ the resistance of a film is given by the Halperin-Nelson (HN) formula\textsuperscript{6,8}

$$R_{HN} = 10.8bR_{n,f} \exp \left[-2\sqrt{b(T_{c0} - T_{BKT})/(T - T_{BKT})}\right] \tag{1}$$

where $R_{n,f}$ is the normal state resistance of the film and $b$ is a non-universal constant. Note that $R_{HN}(T_{BKT}) = 0$.

The LAMH theory\textsuperscript{14} applies to narrow superconducting channels in which thermal fluctuations can cause phase slips, i.e. jumps in the phase difference of the order parameter by $2\pi$. Thermal activations of the system over a free energy barrier $\Delta F$ occur at a rate given by $\Omega(T)e^{-\Delta F/kT}$, where the attempt frequency $\Omega(T) = (L/\xi(T))(\Delta F/kT)^{1/2}\tau_{GL}^{-1}$ is inversely proportional to the relaxation time of the time-dependent Ginzburg-Landau (GL) theory $\tau_{GL} = 8k(T_c - T)/\pi\hbar$ ($L$ is the wire length, $\xi(T)$ is the GL coherence length, and $T_c$ is the mean field critical temperature of the wire).\textsuperscript{11} The free energy barrier for a single phase slip is given by $\Delta F = (8\sqrt{2}/3)(H_c(T)^2/8\pi)(A\xi(T))$, which is essentially the condensation energy density multiplied by the volume $A\xi(T)$ of a phase slip (where $A$ is the wire’s cross sectional area). A current $I$ causes a nonzero voltage (averaged over the rapid phase slip processes)
the occurrence of the BKT transition at this point. Nevertheless, unlike the film sample, the bridge sample shows a nonzero resistance even below the BKT transition. Such resistive tails, occurring at $T < T_{\text{BKT}}$, have been found in all samples with constrictions. Below we analyze these tails in details and demonstrate that they are produced by the phase slip events localized on the constrictions and effectively independent from the adjacent films.

In Fig. 3 the $R(T)$ curves for five samples with bridges are plotted in a log-linear format. The resistance of the sample B1 has been measured down to mΩ range using an LTT. Two transitions are seen in each curve as the temperature decreases. As explained above, the first transition is the BKT transition when the films become superconducting. The second “transition” is in fact the resistive tail mentioned earlier. In order to understand its origin it should be compared to the LAMH theory.

Such comparison is not straightforward since LAMH results apply to long uniform 1D wires, not short bridges. In particular, the attempt frequency $\Omega(T)$ depends on two factors that are not relevant to a bridge. It has a term $L/\xi(T)$ that accounts for the number of independent sites where a phase slip can occur. Since each of our samples has only one narrow spot where a TAPS event can happen, this number equals unity. Similarly the coefficient $\langle DF/kT\rangle^{1/2}$, which takes into account possible overlaps of TAPS at different places along the wire, equals unity for short bridges. So the attempt frequency for a bridge is simply $\Omega_{\text{WL}} = 1/\tau_{\text{GL}}$. Since exact expression for resistance is unknown, we approximate the resistance of a constriction (weak link) as

$$R_{\text{WL}}(T) = R_n e^{-\Delta F_{\text{WL}}/kT}.$$  

The exponential factor here is that of LAMH and the prefactor is simply the normal resistance of the bridge.$^{12}$

<table>
<thead>
<tr>
<th>Sample</th>
<th>$w$ (nm)</th>
<th>$R_N$ (Ω)</th>
<th>$T_c$ (K)</th>
<th>$T_{\text{c0}}$ (K)</th>
<th>$d$ (nm)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>27 ± 4</td>
<td>1380</td>
<td>3.88</td>
<td>3.90</td>
<td>2.5</td>
<td>1.47</td>
</tr>
<tr>
<td>B1</td>
<td>13 ± 4</td>
<td>1650</td>
<td>4.80</td>
<td>4.91</td>
<td>3.5</td>
<td>0.723</td>
</tr>
<tr>
<td>B2</td>
<td>28 ± 4</td>
<td>1320</td>
<td>4.81</td>
<td>4.91</td>
<td>3.5</td>
<td>1.10</td>
</tr>
<tr>
<td>C1</td>
<td>13 ± 4</td>
<td>1440</td>
<td>5.16</td>
<td>5.50</td>
<td>4.5</td>
<td>0.653</td>
</tr>
<tr>
<td>C2</td>
<td>27 ± 4</td>
<td>680</td>
<td>5.39</td>
<td>5.50</td>
<td>4.5</td>
<td>2.21</td>
</tr>
</tbody>
</table>

TABLE I: Sample parameters, including the width of the constriction ($w$), normal resistance of the bridge ($R_N$), critical temperature (determined from $R_{\text{WL}}(T)$) of the bridge ($T_c$), critical temperature of the film ($T_{\text{c0}}$), film thickness ($d$), and a geometrical fitting parameter ($\beta$).
The Eq. 2 allows a simple interpretation: The duration of a single phase slip (i.e., the time it takes for the order parameter to recover) equals $\tau_{GL}$, and the number of phase slips occurring per second is $\Omega_{WL}(T) \exp[-\Delta F(T)/kT]$, with $\Omega_{WL} = 1/\tau_{GL}$. Therefore the fraction of time in which the constriction is populated with a phase slip (i.e., when the gap is suppressed) is the product of these two values, i.e., $f = (\tau_{GL})/(\tau_{GL}) \exp[-\Delta F(T)/kT] = \exp[-\Delta F(T)/kT]$. Following Little, it can be assumed that the bridge has the normal state resistance $R_n$ during the time when a phase slip is present. The resistance is zero otherwise; thus we arrive at the averaged resistance $R = fR_n + (1-f) \times 0 = R_n \exp[-\Delta F/kT]$ as in Eq. 2.

In order to compare Eq. 2 with the experimental results, an explicit expression for $\Delta F_{WL}(T)$ is required. Starting with the usual form derived for a long 1D wire, $\Delta F_{LAMH}(0) = (8\sqrt{2}/3)(H_c^2(0)/8\pi)A\xi(0) = 0.83kT_cR_nL/R_n\xi(0)$ where $L$ is the length of the wire, and using $R_n = \rho_n L/A$, the free energy barrier for a weak link is $\Delta F_{WL}(0) = (8\sqrt{2}/3)(H_c^2(0)/8\pi)\beta wd\xi(0) = 0.83kT_cR_n\beta wd/\rho_n\xi(0)$, where $w$ is the width of the bridge, $d$ is the film thickness, and $\rho_n$ is the normal resistivity. The parameter $\beta$ measures the ratio of the phase slip length along the bridge to the length of a phase slip in a 1D wire. Finally, assuming the same temperature dependence of the barrier as in LAMH, i.e., $\Delta F(T) = \Delta F(0)(1 - T/T_c)^3/2$, we arrive at the expression for a constriction resistance induced by thermal fluctuations as

$$R_{WL}(T) = R_n \exp \left[ -0.83 \frac{\beta wdR_n}{\rho_n \xi(0)} \left( 1 - \frac{T}{T_c} \right)^{3/2} \left( \frac{T}{T_c} \right) \right]$$

The fits generated by Eq. 3 are shown in Fig. 3 as solid lines. A very good agreement is found for all five samples. In particular, sample B1 measured with an LTT, shows an agreement with the predicted resistance $R_{WL}$ over about seven orders of magnitude. Only two fitting parameters are used: $\beta$ and $T_c$ (listed in Table I). The other parameters required in Eq. 3, including $R_n$, $d$, $w$, $\xi(0)$, and $\rho_n \approx 180 \mu\Omega$ cm are known. Such good agreement confirms the expectation that the dissipation in a thin film with a constriction at $T < T_{BKT}$ is solely due to thermal activation of phase slips localized on the constriction. As expected, $\beta \approx 1$ for all samples. An observed trend, however, is that larger values of $\beta$ are found for wider constrictions.

We now discuss nonlinear properties of films with constrictions. Measurements of the differential resistance versus DC bias current $dV/dI$ are shown in Fig. 4. Using these results it is possible to distinguish between the BKT mechanism, which leads to a power-law $V(I)$ dependence, and the LAMH mechanism, which gives an exponential $V(I)$. From Fig. 4 it is clear that an exponential rather than a power law dependence characterizes samples with constrictions (at $T < T_{BKT}$ and sufficiently low currents). Thus it is appropriate to compare the measurements with the LAMH theory which predicts $dV/dI = R(T) \cosh(I/I_0)$, where $R(T)$ is the zero-bias resistance. Using this relation, we fit the differential resistance data as shown in Fig. 4 for different temperatures. A good agreement is observed, confirming that the dissipation is due to LAMH type phase slippage events occurring in the bridge. No indication of enhanced VAP breaking near the bridge is found.

It is remarkable that by fitting high bias $dV/(dI)$
fit obtained from Eq. 3. An excellent agreement is seen was systematically applied to sample B2 (Fig. 5). Fig. 5 curves with a simple LAMH expression \( dV/dI = R(T) \cosh(I/I_0) \). The solid and the dashed curves give the best fits generated by the \( R_{WL}(T) \) \( (T_c = 4.81 \text{ K}) \) and \( R_{LAMH}(T) \) \( (T_c = 5.38 \text{ K}) \) formulae respectively.

curves using \( R_{LAMH} \) (discussed above) can be also used to fit the data. The fit (dashed curve in Fig. 5) demonstrates a good agreement with experiment as well as with the \( R_{WL} \) fit. The only problem is that the usual form of LAMH (dashed curve) gives an unreasonably high critical temperature for the bridge, \( T_c = 5.38 \text{ K} \) in this case, which is even higher than \( T_{c0} = 4.91 \text{ K} \) for the film itself. Note that the original LAMH theory gives \( \Omega(T_c) = 0 \) and consequently \( R_{LAMH}(T_c) = 0 \) \( (\text{Fig. 5, dashed line}) \), which is unphysical. On the other hand, the \( T_c \) extracted from the fits with Eq. 3 are almost equal and slightly lower than the film \( T_{c0} \) (Table I) as expected. Thus our modifications made to the LAMH formulae appear justified for short bridges. We remark that some of measured bridges were wider than \( \xi(0) \), yet the TAPS model produces good fits. This is because the criterion for a superconducting channel to be regarded as one-dimensional is that its width must be \( w \lesssim 4.4\xi(T) \) \( (\text{Ref. 12, p. 510}) \), which is true for all our samples.

In conclusion we have shown that the LAMH theory, with slight modifications, describes the behavior of short and rather wide bridges. Good agreement between the theory and the experiment is observed over eleven decades of resistance. A contribution of VAPs is observed only above the BKT transition temperature in thin film electrodes. No amplification of vortex-antivortex pair fluctuations in the vicinity of the bridge was detected. A new method of resistance determination is suggested.

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\[ R(T) = R(T_c) \cosh(I/I_0) \]

\( R(T) \) measured directly (open circles) and the resistance determined as the best fitting parameter to the \( dV(I)/dI \) curves (solid circles). These two sets of data agree with each other. The solid curve in Fig. 5 is a \( R_{WL} \) fit obtained from Eq. 3. An excellent agreement is seen in a wide range of resistances spanning \( \text{eleven orders of magnitude} \). This re-confirms that the TAPS mechanism is dominant in the bridge samples for \( T < T_{BKT} \). The usual LAMH expression \( R_{LAMH} \) (discussed above) can also be used to fit the data. The fit (dashed curve in Fig. 5) demonstrates a good agreement with experiment as well as with the \( R_{WL} \) fit. The only problem is that the usual form of LAMH (dashed curve) gives an unreasonably high critical temperature for the bridge, \( T_c = 5.38 \text{ K} \) in this case, which is even higher than \( T_{c0} = 4.91 \text{ K} \) for the film itself. Note that the original LAMH theory gives \( \Omega(T_c) = 0 \) and consequently \( R_{LAMH}(T_c) = 0 \) \( (\text{Fig. 5, dashed line}) \), which is unphysical. On the other hand, the \( T_c \) extracted from the fits with Eq. 3 are almost equal and slightly lower than the film \( T_{c0} \) (Table I) as expected. Thus our modifications made to the LAMH formulae appear justified for short bridges. We remark that some of measured bridges were wider than \( \xi(0) \), yet the TAPS model produces good fits. This is because the criterion for a superconducting channel to be regarded as one-dimensional is that its width must be \( w \lesssim 4.4\xi(T) \) \( (\text{Ref. 12, p. 510}) \), which is true for all our samples.

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